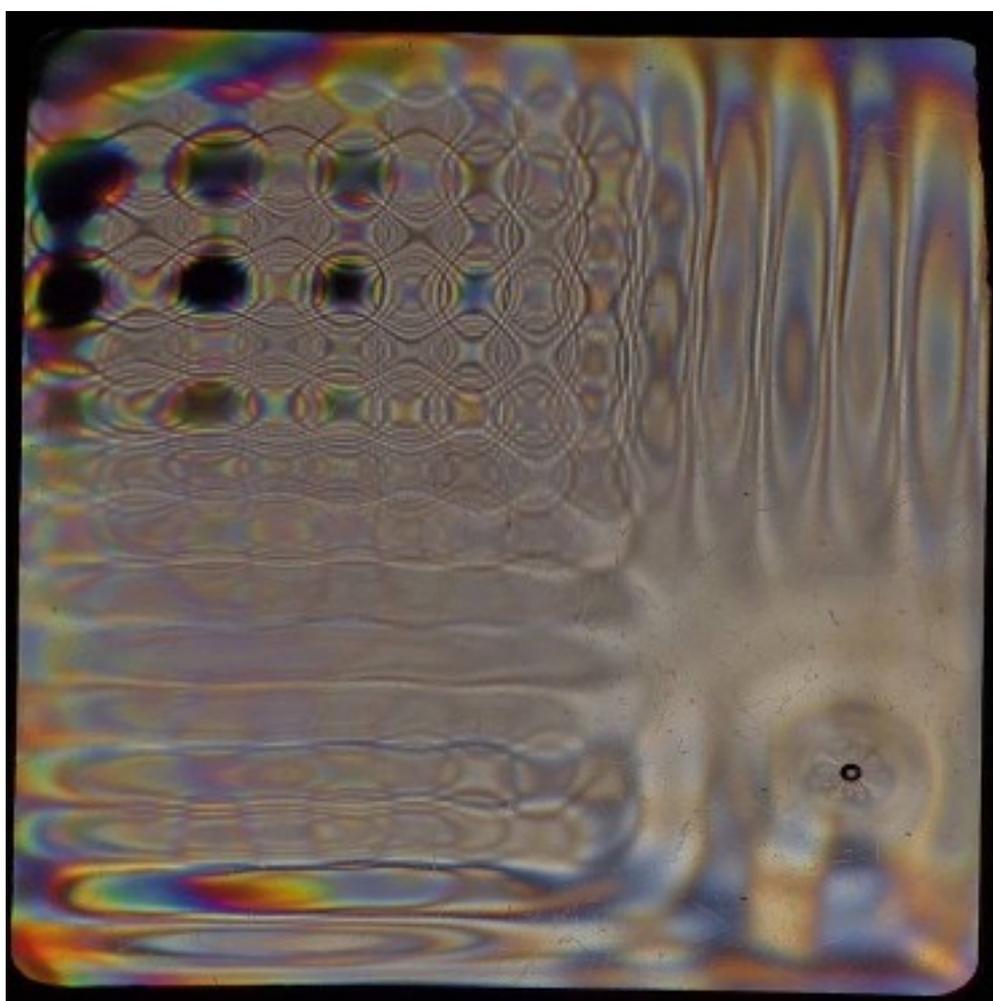


Natural Metrics

The natural way to Minkowski spacetime:
for those who needed a more beautiful,
fully mathematical way to metrics for physics.



Corrado Campisano
Rome, September 2016.
V 0.0.5 – 2016-09-30

Faraday instability, notice the 4 quadrants:
– *upper left*: a crystal-like pattern
– *upper right and lower left*: linear pattern
– *lower right*: below Faraday threshold, with a droplet

(from dotwaves.org)

*There are more things in heaven and earth,
Horatio,
than are dreamt of in your philosophy.*

(William Shakespeare)

*There is only one thing... you can call it vacuum energy,
or you can call it cosmological constant, it's all the same thing.
And if you want to mimic it, to model it with Newtonian physics,
then you do so with a uniform mass [energy] distribution*

(Leonard Susskind)

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Abstract

Mathematicians call it a “[pseudonorm](#)” or “[seminorm](#)”, within their theories of [measurement](#) and [distance](#): it's just [a norm, a metric](#), which has a little more “relaxed” axiom about the “[identity of indiscernibles](#)”.

Physicians call it an infinitesimal “[proper time](#)” or “[proper distance](#)”, depending on the sign they picked when writing it down, which is somewhat inappropriate.

Anyway, it's the “*metrics*” which defines [Minkowski spacetime](#), the one introduced by Einstein in 1905 with Special Relativity, which still sound too strange, to too many people:

$$ds_m^2 = dt^2 - (dx^2 + dy^2 + dz^2)$$

The perceived strangeness being in that minus sign and in the relation with the *distance definition* (the “metrics”) in ordinary [Euclidean space](#), which is by good old [Pythagoras's theorem](#):

$$ds_e^2 = dx^2 + dy^2 + dz^2$$

This paper shows [a new way to get to](#) Minkowski spacetime definition:

- it's the *most natural* way to actually think about, design and engineer metrics for physics, i.e. the foundations of most physical “measurements” and “experiments”, given it defines “how to measure” spacetime.
- it's the *most mathematical* way too, as it's in the mathematical nature of numbers, to require irrational numbers for distances in space, and imaginary numbers for distances in time.
- it's also *physically sound*, as it makes quite “natural” - if not “fully legitimate” - for physicians to keep using their strange mathematical trick, when actually calculating (walking the walk, not talking the talk) their “Feynmann's path integrals”.

In this paper, Pythagoras's “*number is everything*”, Galileo's “*Nature is written in Maths*” and the “[unreasonable effectiveness of Mathematics in the Natural Sciences](#)” are taken quite seriously, and with a proactive attitude.

Indeed, the approach here would be “*MBMU*”: *better include the Most Berserk Maths Upfront*, facing the monster when it's just a kid, rather than later on, with the full grown up monster ready to kick hard, just like getting infinite, non converging integrals, for example.

Finally, this paper is a sort of follow up to “Monkey Metrics”: that had been written after having taken all the “core” courses, from “The Theoretical Minimum”; this one comes after having taken all the “supplemental” courses too, where I found that what I called “no-ether” is also called “dark energy”, or “gravitational constant” or “vacuum energy”: an energy distribution which picks no preferred reference frame. But that's for the physical implementation. [[SHOULD THIS BE HERE?](#)].

A clear trend in history

I studied history of Maths on [Morris Kline's](#) “Mathematical Thought From Ancient to Modern Times” (“Storia del pensiero matematico occidentale”, in italian), but [Wikipedia's page on the subject](#) would be enough to show the point: **new and stranger “kinds of number”** were required, for doing more complicate Maths, during history, along with the growing of knowledge.

Also in physics it's like that, as you can check on the “[Theoretical minimum](#)” by Leonard Susskind: [Quantum Field Theory](#) today needs “*anti-commuting*” numbers or [Grassmann numbers](#) to work.

Notice sometimes those “**strange new numbers**” got “repudiated” by their own “inventors”, the [Pythagoreans](#) being the most relevant example, as, according to the mentioned Wikipedia page:

The study of mathematics as a demonstrative discipline begins in the 6th century BC with the Pythagoreans, who coined the term "mathematics" from the ancient Greek μάθημα (mathema), meaning "subject of instruction".

And, further on, but only on the [italian page on the subject](#), which is more detailed on the “sectarianism” of the “community”:

Paradossalmente la scoperta più importante della comunità fu forse la dimostrazione che il rapporto tra il lato e la diagonale di un quadrato (ossia [radice di 2](#)) non è esprimibile come rapporto di due interi. Questa scoperta, che prova l'esistenza dei [numeri irrazionali](#), si scontrava con tutta la filosofia della setta. Secondo la tradizione riportata da alcuni autori posteriori, il pitagorico [Ippaso di Metaponto](#) fece tale scoperta durante un viaggio in nave, ed ebbe l'infelice idea di comunicarla senza indugio agli altri adepti della setta, i quali comprendendone immediatamente le conseguenze gettarono lo stesso Ippaso in mare. Altri autori menzionano semplicemente il fatto che Ippaso morì in un naufragio. Di fatto, se pure ci fu un tentativo dei pitagorici di tenere nascosta la scoperta, questo non riuscì.

That may sound weird, but it's not for weird mathematicians only: even physicians happened to “repudiate” their own “creations”.

For example, Einstein really gave birth to Quantum Mechanics, with his 1905 paper “[On a Heuristic Viewpoint Concerning the Production and Transformation of Light](#)”, and [won the Nobel for it](#), not for Special Relativity, or for any other of the [1905 papers](#).

But then [he didn't look to like QM so much](#), or rather at all, to the point of going mystical with “*Subtle is the Lord...*” and of getting into his final “[Waterloo](#)” with the “[EPR paradox](#)” and the “*spooky action at a distance*”.

Knowing history is worth for avoiding redoing mistakes: it applies to Maths and Physics too.

Primitive maths: from naturals to rationals

Primitive Maths was “*invented*”, not “*discovered*” (indeed it'd rather more correct to say that the “*maths' memes emerged*”, but that's OT), for several practical applications, the main ones being:

- **counting** (children, preys, predators, enemies);
- **accounting** (reserves for wintertime, but also stones and clubs [first](#), bows and arrows [later](#));
- **trading** (exchanging goods).

That had been done mostly by all ancient civilizations, independently.

First numbers to be “invented” were the ones for counting, i.e. [natural numbers](#): how many sheep or how many apples.

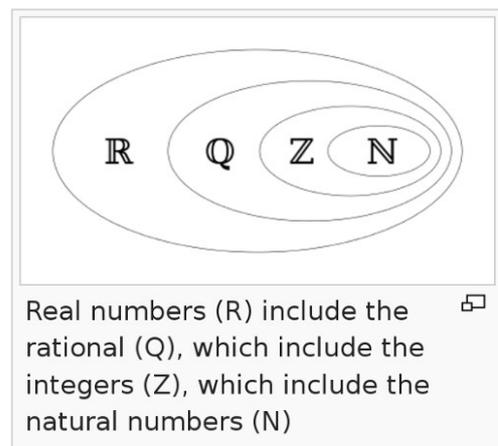
Then [rational numbers](#) entered the game, for trading: how many apples for one sheep.

Integers would be late, sorry

That sequence (natural then rational) was somewhat “*inappropriate*”, since, from a [set theory](#) perspective, in the “Number System” (or in the “[classification of numbers](#)”), right “after” the natural numbers, the [integer numbers](#) “should come in next”, not the rational ones:

Main number systems

\mathbb{N}	Natural	0, 1, 2, 3, 4, ... or 1, 2, 3, 4, ... \mathbb{N}_0 or \mathbb{N}_1 are sometimes used.
\mathbb{Z}	Integer	..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...
\mathbb{Q}	Rational	$\frac{a}{b}$ where a and b are integers and b is not 0
\mathbb{R}	Real	The limit of a convergent sequence of rational numbers
\mathbb{C}	Complex	$a + bi$ where a and b are real numbers and i is the square root of -1



Probably that's because at that time accounting was not so sophisticated and there was only “trade”, not “finance” too, meaning they only had to track what they had, for actual trade: there was no “shorting” option available then.

Indeed, the “**zero**” was *not known* by the “*Greek inventors of deductive Maths*” and the **negative numbers** (and hence the integer numbers) were “[not well accepted till Renaissance](#)”:

Nell'[Europa](#) del cinquecento, e in particolare in [Italia](#), si diffuse un forte interesse per l'[algebra](#). In questo secolo si cominciarono ad accettare i [numeri negativi](#) chiamati spesso "falsi".

Noticeably, the Renaissance also gave birth to financial Maths, especially on accounting, with the “[double entry book-keeping](#)” (where “assets” are positive and “liabilities” are “conceptually” negative), but also with [good improvements in algebra](#) (solution of quartic equation, imaginary quantities, decimal notation, etc).

Indeed, Renaissance people took a lot from the “Hellenistic” or “Alexandrian” mathematicians and scientists, who were a lot less scary:

[Heron of Alexandria](#) (c. 10–70 AD) is credited with [Heron's formula](#) for finding the area of a scalene triangle and with being the first to **recognize the possibility of negative numbers possessing square roots**.

But that's a lot later, than the “primitive maths” of natural and rational numbers.

Rationals not so good for geometry, sorry

After having invented natural and rational numbers, the “*primitive mathematicians*” started using rationals even for measuring farming fields, which was mostly critical in Egypt, because of the Nile behaviour and the subsequent [yearly need to “redraw the borders”](#) of anyone's fields.

Indeed, they mostly ended up writing practical [Pythagorean triples](#) tables, just like mathematicians and scientists kept writing tables for trigonometry and logarithms, until calculators came in.

For sure they knew something about the [Pythagoras's theorem](#), as [amongst the hundreds of way to demonstrate it](#), there is one, experimental-like, very closely related to the flooding: just count the buckets of water it would take to cover the three areas with water 1cm deep. Details in Appendix.

Probably they didn't know – or just didn't care a lot – about one subtle problem with space: **measuring space is not really the same of counting apples**.

It's **not even like** saying that distance is “*some proportion of a sample unit length*”, as the Pythagoras's theorem itself brings in the problem of “[incommensurable lengths](#)”, which is the subject of the next chapter.

Key concepts from primitive Maths

Worth listing them explicitly:

1) the various “**kinds of numbers**”, and hence their own “properties”, including any “strangeness”, are, as [Algebra or the “rules of putting things together”](#) show, consequences of the properties assigned to operations defined on (unary) and/or between (binary) them, while keeping the system closed (results of operations must still be of that kind of number, which is the “[closure rule](#)”).

1-bis) *violating the closure rule is bad*

2) not all kind of numbers are **appropriate to perform different activities**: to count apples natural numbers are enough, but to sum apples to oranges (like in an exchange, with the sum being zero), you will need a rational number to provide a “[ratio](#)”, to “weight” one apple to one orange with.

2-bis) *summing up apples with oranges is bad*

Pythagoras: from rational to irrational numbers

The “Pythagoreans” were the first ones to say “*all is number*” or something like that.

Indeed they had great theoretical troubles, in their “theory”, with the square root of 2.

It was “incommensurable”, i.e. not comparable, like they were used to do, knowing rational numbers only, to any “unit”. Take a square with your favorite unit as a side, or a rational value of that unit: the diagonal it's not “commensurable” to that unit, i.e. there is no rational number for that.

They were able to “prove” this by “[reductio ad absurdum](#)”, playing around those numbers being odd or even, but they couldn't get to the idea that those numbers could had been just “not appropriate”.

Measuring space needed real numbers, rational ones were not enough: unfortunately the root of their “beliefs” was that “*all is (rational ***) numbers*”, so they remained stuck in that “dead end”.

It was a great shock for them, as they proved the Pythagoras theorem, from which that square root of 2 was coming. Not so far from what Einstein's feelings for QM could had been: sort of “[tu quoque, Brute, fili mi](#)”.

So they, as any “respectable” sept would have done, kept that “little trouble” as secret as they could, just like proprietary software houses do with the bugs in their code: hide it, deny it, whatever to keep it secret.

People who care too much about their reputation have none, to me, and less than zero those who won't admit a mistake. Mistakes are where new information is hiding, so let's try understand what the problem was.

Note (*)**: it was actually integer numbers, they were aware they could “rescale” any sets of rational numbers into integers.

Rationals, resolution and space dimensions

Primitive mathematicians were quite fine with using rational numbers for geometry, as mostly all their maths was approximated (with the noticeable exception of the Egyptians with the volume of pyramids). Even the mathematical notation was primitive, mixing different ones, using a lot of fractions (rationals).

Even with the most exact formulas, they could not have noticed the “infinite resolution” problem (the density of \mathbb{Q} in \mathbb{R}) that's working in the background, as that's not something which becomes evident until you get to the Pythagoras's theorem. You also need to use a “theoretical” approach, like the Greeks started to do, not the “empirical” ones of the “primitives”.

First thing to notice, is that you need to be working with **at least two space dimensions**. With one space dimension, and finite resolution, rational numbers can work pretty fine. Little (if not none) ways to figure out the “comb” between \mathbb{Q} and \mathbb{R} [**FIND A LINK, see Avvantaggiati's book**].

When you “add” distances along the same direction, it's a basic sum, just like when adding apples to apples.

It's when you want to “add” a distance, along a direction, with a distance, along another (orthogonal) direction, that it seems more like summing apples to oranges. Even stranger than that.

Experimental evidence first (for the primitive mathematicians) and theoretical reasoning later (for the Greeks) were the “proof” that distances along different directions are not directly summed: you will need to sum their square, and then take the square root.

Quite more complicated that the “ratio”-weighted sums they were used to do with apples and oranges.

The ancient Greeks' pitfall and ruin...

It's what came after Pythagoras and the Pythagorean school: [Zeno's paradoxes](#) (all non-paradoxes, once one has an understanding of real numbers, quite like with Godel's “inconsistency theorem”, by the way) and the Eleatic school first, and the final decline (philosophically, at least) with the [Sophists](#) later.

All Zeno's paradoxes were about the ***difference between discrete and continuum***: as they were using rationals for space, ***inappropriately***, they got into paradoxes. Yet Achilles could overrun the tortoise, the arrow hit its target and so on.

Experimentally falsified abstract reasoning, that's it: down into the trash bin of science history.

But, for a strange revenge of Nature, long later on, the ***discrete vs. continuum*** still debate goes on, with Einstein having been in the eye of the hurricane:

- 1905's “Brownian motion” paper says spacetime is continuum, matter is discrete;
- 1905's “Photoelectric effect” paper says the same, but applied to “energy” rather than matter;
- 1922 speech in Leiden recognizes, for both GR and SR, the need for a “continuum” on which to do “continuum mechanics” on - but all “continuums” are actually “made up of” discrete units (from water molecules for fluid-dynamics, to virtual particles and oscillators for vacuum energy);
- nowadays “relativists” (the ones, in the physics “community”, who'd rather stand for GR, than for QM) had evolved into “Quantum Loop Gravity” theorists, who says “space is discrete and time does not exist”, going back to Zeno's paradoxes. I tried to tell Rovelli about this, but he didn't get the point, when he published “La realtà non è come ci appare”.

Quite messy, indeed.

Enough for the suspect they're just doing something inappropriate, like the Pythagoreans with the rational numbers.

Noticeably, anyway, nowadays “quantists” (the ones, in the physics “community”, who'd rather stand for QM, than for GR) had evolved into “Quantum Field” theorists first, and “String” theorists later: they start with complex numbers for their “system states”, and let in very strange new kind of numbers along the path. But they too have some paradoxes and problems with experiments.

Enough with the diversions: let's go back to metrics for spacetime.

Einstein: from real to imaginary numbers

It's not just the metrics of Special Relativity, which has time behaving as if it were to be considered imaginary, as suggested by the metrics definition and detailed in the following chapter.

It's also the ubiquitousness of the imaginary unit in Quantum Mechanics, Einstein's repudiated child, that is quite impressive.

Every time “time” is involved, there's an “i”, somewhere near that “t”, in the equations:

$$H = -i\hbar\partial/\partial t \quad \overrightarrow{q \cdot \text{state}} = e^{(ipx)} \quad \overrightarrow{q \cdot \text{state}} = e^{(iEt)}$$

[**CHECK, maybe LIST**], let alone the *mandatory* “strange trick” with Feynmann's path integrals.

Indeed, here we will try to define metrics for spacetime, i.e. the rules on “how to add” space distances with time distances and get something meaningful out.

But that would be in the next chapter, let's first disclose a couple of “dirty tricks” Einstein used for supporting his “no ether needed”, kinematical workaround, described in “Monkey Metrics”.

Is it “proper what”, please?

The first thing to be “suspect”, to make anyone's eyebrow rise, is that strange attitude of calling the same thing two different ways, when a minus sign is applied.

Indeed, from Einstein on, this is called “**proper time**”:

$$d\tau^2 = dt^2 - dx^2$$

while this is called “**proper distance**”:

$$ds^2 = dx^2 - dt^2$$

while, of course:

$$d\tau^2 = -ds^2$$

Why do they do this? So that that thing, which is the square of something, stays positive.

Mathematicians should kick Physicians hard in their asses, for that: “It's a *pseudo-norm*, you idiots, it's a '*seminorma*', that's why it can get negative. Check out the definition: that grants it the 'right to be negative'... and the definition will also tell you the actual physical meaning of that, if you are able to read it. But you are his worshipping followers, and he was bad in Maths, so...”

That may sound arrogant, but that comes with the bundle: “[as much baloney as you can](#)” was the punchline in that TED talk.

And, I warned you: here we're taking the “[unreasonable effectiveness of Mathematics in the Natural Sciences](#)” very very very seriously.

So remember: *Most Berserk Maths Upfront.*

Hidden in the “elegant” notation

Another very “suspect” place for “the imaginary nature of time” to be hidden, is the [covariant and counter-variant notation](#), introduced by Einstein, for his 4-vectors of spacetime.

Indeed, in a [Minkowski spacetime](#), if a covariant [4-position](#) is given by:

$$X_\mu = (-ct, x, y, z)$$

then its counter-variant version is:

$$X^\mu = (ct, x, y, z)$$

What for? So that its “length” is:

$$|X| = X_\mu X^\mu = x^2 + y^2 + z^2 - c^2 t^2$$

Wow, that looks like shooting a fly with a bazooka...

What about something simpler like this, then?

$$X_i = (ict, x, y, z)$$

It's not a four-vector, it's just a “*vector of four mixed components*”, three real and one imaginary; its length is still:

$$|X| = X_i X_i = x^2 + y^2 + z^2 - c^2 t^2$$

Honestly, I'm not sure I could be able to “rewrite” all the Maths, where 4-vector are used, in terms of these “*vectors of four mixed components*”, but that could be just me not smart enough with Maths.

Still, I'll follow my intuition, as history is supporting it: this really smells like “*time is still suspect*”.

Maybe it's just history repeating?

Measuring spacetime needs imaginary numbers, just like measuring space needed real numbers.

Today that's not kept so secret, as it was for Pythagoreans: you only have to spend a few hundreds hours watching Leonard Susskind introduce most of the physics we know today, before you get the suspect something similar (a real need for different numbers) it's happening today.

Still, it's not only not clearly taught that way, maybe because it had not been clarified yet.

In the following chapter, after a recap of the definition of distance in space, to be used as reference, I'll try to introduce **two “missing descriptions” of “distance in spacetime”**, which clearly emerge by manipulating a little the “metrics definition” we started with.

Again, it's Maths ruling, I'll let its “autopilot” bring me to the solution, like Susskind liked to do.

A new way to Minkowski spacetime

No smoke and mirror, let the equations talk.

Defining distance in space

Distances in space are defined as follows, by the “spatial distance”, or the metrics:

$$ds^2=(x, y, z)(x, y, z)=dx^2+dy^2+dz^2$$

This is nothing else than the Pythagorean theorem applied to an infinitesimal distance.

Defining distance in spacetime

Let's start saying distances in spacetime ***cannot be*** defined as follows, with the same “Pythagorean distance”, or the metrics:

$$ds^2=(x, y, z, t)(x, y, z, t)=dx^2+dy^2+dz^2+dt^2$$

That's simply because ***time is not space***, you can't “add” time distance to space distance, like you added, for example, the z distance to the x,y distance, when *extending* the definition from 2d to 3d.

Also, it would be ***logically meaningless***, as with ***a question like***:

“what's the distance between (Rome, today) and (Paris, tomorrow)?”

will only get ***a somewhat “relative answer”***, like this:

“it's a distance you can cover with an airplane,
while with a train maybe you'd need 2 days...
so don't even think about driving there!”

I've been working in the [travel industry](#), for a primary [GDS](#) for 11 years, I know what I mean.

Do you?

I'll let Maths talk now: two simple manipulations, and you get the two ideas above in terms of equation to look at.

Mathematical meaning

The first algebraical manipulation is quite simple, almost too simple to mean anything.

Let's start with this formulation:

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2$$

The easiest, stupidest and most basic, algebraical manipulation you can do is this:

$$ds^2 = dx^2 + dy^2 + dz^2 + (i dt)^2$$

Or, in the scalar product form:

$$ds^2 = (x, y, z, it)(x, y, z, it) = dx^2 + dy^2 + dz^2 - dt^2$$

Time behaves, for the metrics, as it were “pure imaginary”, so that it's still a “Pythagorean distance” being defined, but using “some new kind of numbers”.

Notice *it's not the distance in the complex plane*, which is defined very differently, in terms of the complex conjugate:

$$ds^2 = (x, iy)(x, -iy) = dx^2 + dy^2$$

while here, reducing to x,t, it is:

$$ds^2 = (x, it)(x, it) = dx^2 - dt^2$$

So Maths says:

- if space is “real”, time has to be “imaginary”;
- this with the constraint that (x,t) is not the complex plane.

But if so, what's this (x,t) plane then?

it's 1D spacetime, something midway between 2D space and the complex plane.

Full stop: it's the equations saying that, it's not my fault, sorry.

It's history repeating, it's not my fault, if that's sound berserk.

Physical meaning

Yep, the “physical meaning” of a distance in spacetime is expected to be meaningful.

But we'll get there with another, simple algebraical manipulation, which also requires a geometrical and physical interpretation, which is exactly the “travel industry example” above.

Again, let's start with this formulation:

$$ds^2 = dx^2 + dy^2 + dz^2 - dt^2$$

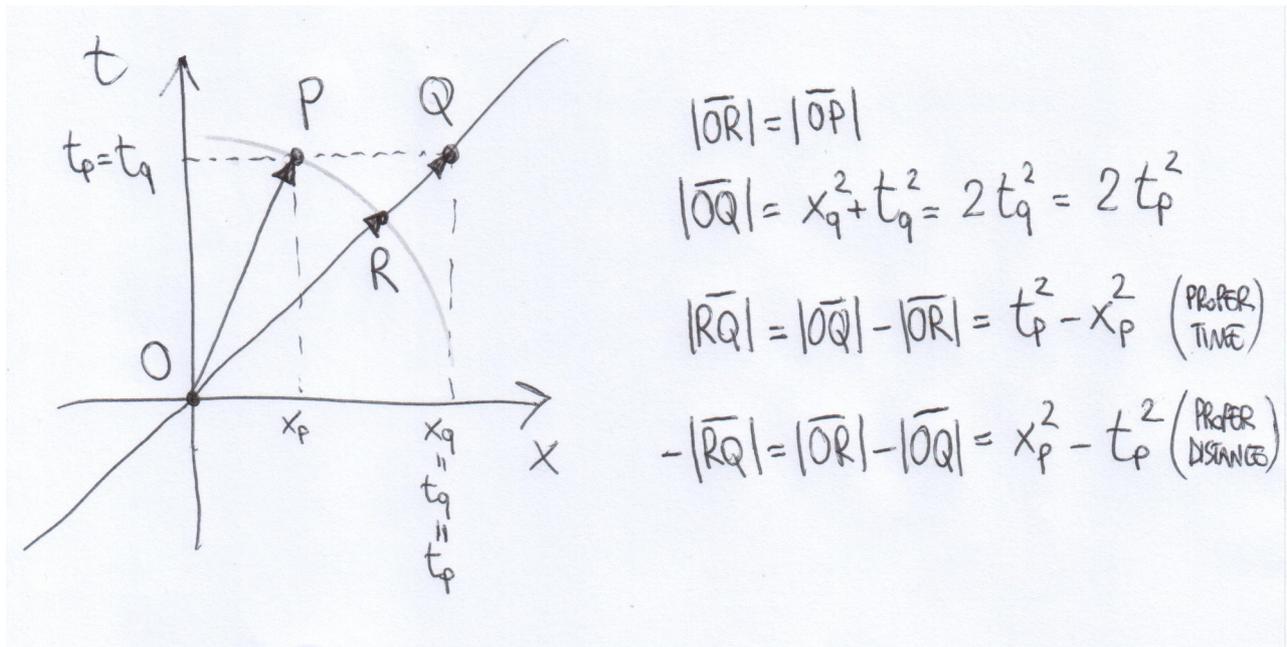
And reduce to x,t for simplicity:

$$ds^2 = dx^2 - dt^2$$

The algebraical manipulation is writing it as:

$$ds^2 = dx^2 + dt^2 - 2dt^2$$

so that this “spacetime distance” could be interpreted as the difference of two “Pythagorean distances” on the (x,t) plane:



But then... $|OP|$ is the length of the trajectory (the OP segment) of P moving from O to P, the same for $|OQ|$.

So then the “spacetime distance” is the difference between those two Pythagorean distances in the x,t plane.

That's because you need a “reference motion” to define time, as shown in Monkey Metrics.

And that motion shall have a finite “speed”, being that “1” or “c”, depending which units you work with.

It could had been the speed of sound, if we happened to evolve from bats, instead than from rats.

Hopefully, we (monkeys) have the “vacuum energy”, or the “cosmological constant” or the “dark energy”, to provide for some “background engine” producing a “reference motion” that “has the same speed in all inertial reference frames”, and doing so “without picking up a preferred reference frame”, ipse dixit Leonard Susskind, in one of his last lectures of the “Theoretical Minimum”

[REVISE, EXEND]

Appendix

A few side-by arguments are collected here.

Bricks and Mortar Pythagoras

Every worker in the construction industry should be able to demonstrate the Pythagoras's theorem, experimentally, by using only “traditional” tools and materials.

Indeed, ancient Egyptians and Babylonians had no “proof” of the theorem, but they used it a lot, mostly by approximation or interpolation with the “Pythagorean triads” tables, like it had been done with logarithms and trigonometry for a long long time, till calculators were available.

So, how could they “trust” something they couldn't prove?

It should have been some “fact of nature” they could have taken as “experimentally true” and “self evident”. Incidentally that's the same way “principles” are picked in Physics.

Indeed, both civilizations were quite good in dealing with bricks and mortar, especially for “governing” water: the following “experimental proof” is all around that.

It will also be described as they did: “recipes”, or algorithms, to get to the solution.

It's just geometry and physics

Nail down, on a plain ground, two sticks, at a distance of your choice. That will be your first “leg” (“cateto”). Choose one of the two sticks, to be the one where the “orthogonal” angle will be: call it A and B the other one. With a rope, draw on the ground the line connecting A and B, extending the line beyond A, for a distance larger than the distance from A to B. Then nail the rope in A and use it to find the point C, being at the same distance from A as B is, over the line, on the other side of B, where the line was “extended”. Now nail the rope in C first and B then, taking a distance, on the rope, larger than the distance from A to B; do so to draw two semi-circles on the ground, so that they will be “around” A. Those two semi-circles would cross in two points, call them D and E. Draw the line connecting D and E: it will pass through A too. Now nail down a stick on this line, call it point F, choosing any distance you like from point A. That would be your second leg. The “hypotenuse” will be the segment connecting B and F. You now have a “square triangle”.

Now build three square pools, of the same height, taking care they have uniform depth, along the three segments AB, DF and BF.

Finally, count down the bucket of waters you will need to fill the three pools: you'll find that the number of buckets, needed to fill the square pool along BF, will be the sum of the buckets, needed to fill the other two pools.

Same amount of waters means same volumes, and hence, having all three pools the same and uniform depth, it means same areas:

$$AB^2 + DF^2 = BF^2$$

Vacuum energy as the “no-ether”

Work in progress...

see quotes at the beginning, for now...

Acknowledgments

Of course I could never reach writing this without leveraging on the work of a huge lot of people, which I can hardly list down correctly.

So I'll keep the list very short, tracking just the most relevant people:

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- Morris Kline, for “Storia del pensiero matematico occidentale”
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- professor Carlo Rovelli, for “La realtà non e' come appare”
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